

# Unkonventionelle Supraleitung

## Serie S1

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Multiple superconducting phase transitions.

**S1** We consider a system described by the following Ginzburg-Landau free energy density  $f$  with two-component superconducting order parameter  $\vec{\eta} = (\eta_x, \eta_y)$ .

$$f = a'(T - T_c)|\vec{\eta}|^2 + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\{\eta_x^{*2}\eta_y^2 + \eta_x^2\eta_y^{*2}\} + b_3|\eta_x|^2|\eta_y|^2,$$

where the coefficients  $a' (> 0)$  and  $b_i$  ( $i = 1, 2, 3$ ) are real numbers. we take  $b_1 > 0$ .

Let us investigate the situation that a symmetry reduction of the above system occurs, lifting the degeneracy of the two components  $\eta_x$  and  $\eta_y$ . Then, the second order term of  $f$  changes:

$$a'(T - T_c)|\vec{\eta}|^2 \quad \rightarrow \quad a'(T - T_{cx})|\eta_x|^2 + a'(T - T_{cy})|\eta_y|^2.$$

[See Eq. (4.29) or (181) in the theory lecture notes.] Here, we assume that the degeneracy lifting is small such that  $T_{cx}$  and  $T_{cy}$  are only slightly different, i.e.,  $|T_{cx} - T_{cy}| \ll T_{cx}, T_{cy}$ . We assume  $T_{cx} > T_{cy}$ .

The first superconducting transition occurs at  $T = T_{cx}$ . That is,  $\vec{\eta} = (0, 0) \rightarrow \vec{\eta} = (\eta_x, 0)$  at  $T = T_{cx}$ . Then, the second transition from this phase  $\vec{\eta} = (\eta_x, 0)$  to lower-temperature phase  $\vec{\eta} = (\eta_x, \eta_y)$ , occurs at  $T = T'_{cy} (< T_{cx})$ . It is known that  $T'_{cy}$  is different from  $T_{cy}$ .

Problem: Show that the second transition temperature  $T'_{cy}$  is given by

$$T'_{cy} = \max\{T_{y+}, T_{y-}\},$$

where

$$T_{y\pm} = T_{cy} \frac{1 - R_{\pm} T_{cx} / T_{cy}}{1 - R_{\pm}}, \quad \text{with} \quad R_{\pm} = 1 + \frac{b_3 \pm b_2}{2b_1}.$$

[See Eq. (4.35) or (188).]