Unkonventionelle Supraleitung WS 05/06
Lösungen zur Serie 10

The condensation energy near the critical temperature T_c.

10.1 Let us consider the unitary triplet states $\hat{\Delta}_k = \vec{d}_k \cdot \hat{\vec{\sigma}} i\hat{\sigma}_y$, where $\vec{d}_k \times \vec{d}_k^* = 0$ and $|\Delta_k| = \sqrt{\frac{1}{2}\mathrm{Tr}[\hat{\Delta}_k\hat{\Delta}_k^\dagger]} = |\vec{d}_k|$. We define the amplitude $|\eta| \equiv \sqrt{\left\langle|\vec{d}_k|^2\right\rangle_k}$ and the normalized d vector $\hat{d}_k \equiv \vec{d}_k/|\eta|$. Thus, $\left\langle|\hat{d}_k|^2\right\rangle_k = 1$. Here, $\langle\cdots\rangle_k$ denotes the average over the spherical Fermi surface, $\langle\cdots\rangle_k = \int \frac{d\Omega_k}{4\pi}\cdots$.

a) The Ginzburg-Landau free energy for spatially uniform systems is given as follows *within the weak-coupling BCS theory.*

$$
\begin{aligned}
F &= F_n - \alpha\Big(1-\frac{T}{T_c}\Big)\Big\langle|\Delta_k|^2\Big\rangle_k + \frac{\beta}{2}\Big\langle|\Delta_k|^4\Big\rangle_k && (1)\\
&= F_n - \alpha\Big(1-\frac{T}{T_c}\Big)\Big\langle|\vec{d}_k|^2\Big\rangle_k + \frac{\beta}{2}\Big\langle|\vec{d}_k|^4\Big\rangle_k && (2)\\
&= F_n - \alpha\Big(1-\frac{T}{T_c}\Big)\Big\langle|\hat{d}_k|^2\Big\rangle_k|\eta|^2 + \frac{\beta}{2}\Big\langle|\hat{d}_k|^4\Big\rangle_k|\eta|^4 && (3)\\
&= F_n - \alpha\Big(1-\frac{T}{T_c}\Big)|\eta|^2 + \frac{\beta}{2}\Big\langle|\hat{d}_k|^4\Big\rangle_k|\eta|^4, && (4)
\end{aligned}
$$

where $\alpha = N_0$ (>0) and $\beta = 7\zeta(3)N_0/8(\pi k_B T_c)^2$ (>0). F_n and N_0 are the free energy and the density of states (per spin projection) in the normal state, respectively. $\zeta(3) = \sum_{n=1}^{\infty} n^{-3} \approx 1.2$ is the Riemann zeta function. Below T_c, the above free energy has a minimum value with respect to η, ($\partial F/\partial\eta^* = 0$).

$$
0 = \frac{\partial F}{\partial\eta^*} = -\alpha\Big(1-\frac{T}{T_c}\Big)\Big\langle|\hat{d}_k|^2\Big\rangle_k\eta + \frac{\beta}{2}\Big\langle|\hat{d}_k|^4\Big\rangle_k 2\eta|\eta|^2, \tag{5}
$$

$$
\rightarrow \qquad 0 = -\alpha\Big(1-\frac{T}{T_c}\Big)\Big\langle|\hat{d}_k|^2\Big\rangle_k + \beta\Big\langle|\hat{d}_k|^4\Big\rangle_k|\eta|^2. \tag{6}
$$

From it,

$$
|\eta|^2 = \frac{\alpha}{\beta\Big\langle|\hat{d}_k|^4\Big\rangle_k}\Big(1-\frac{T}{T_c}\Big). \tag{7}
$$

Substituting this $|\eta|^2$ into the free energy, we obtain the condensation energy F_{cond} as

$$
\begin{aligned}
F_{cond} &\equiv F - F_n && (8)\\
&= -\alpha\Big(1-\frac{T}{T_c}\Big)\left[\frac{\alpha}{\beta\Big\langle|\hat{d}_k|^4\Big\rangle_k}\Big(1-\frac{T}{T_c}\Big)\right] + \frac{\beta}{2}\Big\langle|\hat{d}_k|^4\Big\rangle_k\left[\frac{\alpha}{\beta\Big\langle|\hat{d}_k|^4\Big\rangle_k}\Big(1-\frac{T}{T_c}\Big)\right]^2 && (9)\\
&= \frac{-\alpha^2}{2\beta\Big\langle|\hat{d}_k|^4\Big\rangle_k}\Big(1-\frac{T}{T_c}\Big)^2. && (10)
\end{aligned}
$$

b) On the other hand, when taking into account the so-called *feedback effect* in the superfluid ^{3}He, the following term is added to the free energy according to the Leggett's review article.[1]

$$
\begin{aligned}
F_{FB} &= \frac{\beta}{2}\gamma|\eta|^4 \sum_{i,j}\Big[\mathrm{Re}\Big\langle \hat{d}_k^{i*}\hat{d}_k^{j}\Big\rangle_k \Big(\delta_{i,j} - 2\mathrm{Re}\Big\langle \hat{d}_k^{i*}\hat{d}_k^{j}\Big\rangle_k\Big)\Big] && (11)\\
&= \frac{\beta}{2}\gamma|\eta|^4\Big[\Big\langle|\hat{d}_k|^2\Big\rangle_k - 2\Big\{\Big\langle|\hat{d}_k^x|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^y|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^z|^2\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{x*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2 - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{y*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{y*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2 - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{z*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{z*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2 - 2\Big\{\mathrm{Re}\Big\langle\hat{d}_k^{x*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2\Big] && (12)\\
&= \frac{\beta}{2}\gamma|\eta|^4\Big[1 - 2\Big\{\Big\langle|\hat{d}_k^x|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^y|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^z|^2\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{x*}\hat{d}_k^{y} + \hat{d}_k^{y*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2 - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{y*}\hat{d}_k^{x} + \hat{d}_k^{x*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{y*}\hat{d}_k^{z} + \hat{d}_k^{z*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2 - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{z*}\hat{d}_k^{y} + \hat{d}_k^{y*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2\\
&\qquad - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{z*}\hat{d}_k^{x} + \hat{d}_k^{x*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2 - 2\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{x*}\hat{d}_k^{z} + \hat{d}_k^{z*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2\Big], \qquad \Big(\Big\langle|\hat{d}_k|^2\Big\rangle_k = 1\Big) && (13)\\
&= \frac{\beta}{2}\gamma|\eta|^4\Big[1 - 2\Big\{\Big\langle|\hat{d}_k^x|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^y|^2\Big\rangle_k\Big\}^2 - 2\Big\{\Big\langle|\hat{d}_k^z|^2\Big\rangle_k\Big\}^2\\
&\qquad - 4\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{x*}\hat{d}_k^{y} + \hat{d}_k^{y*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2\\
&\qquad - 4\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{y*}\hat{d}_k^{z} + \hat{d}_k^{z*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2\\
&\qquad - 4\Big\{\frac{1}{2}\Big\langle\hat{d}_k^{z*}\hat{d}_k^{x} + \hat{d}_k^{x*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2\Big] && (14)\\
&= \frac{\beta}{2}|\eta|^4\Big[\gamma - 2\gamma\Big\{\Big\langle|\hat{d}_k^x|^2\Big\rangle_k\Big\}^2 - 2\gamma\Big\{\Big\langle|\hat{d}_k^y|^2\Big\rangle_k\Big\}^2 - 2\gamma\Big\{\Big\langle|\hat{d}_k^z|^2\Big\rangle_k\Big\}^2\\
&\qquad - 4\gamma\Big\{\Big\langle\hat{d}_k^{x*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2 - 4\gamma\Big\{\Big\langle\hat{d}_k^{y*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2 - 4\gamma\Big\{\Big\langle\hat{d}_k^{z*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2\Big]. \qquad (\hat{d}_k^* \times \hat{d}_k = 0) && (15)
\end{aligned}
$$

Here, γ is the numeric factor which represents the strength of the feedback effect. Then, the Ginzburg-Landau free energy is given as

$$
\begin{aligned}
F &= F_n - \alpha\Big(1 - \frac{T}{T_\mathrm{c}}\Big)|\eta|^2 + \frac{\beta}{2}\Big\langle|\hat{d}_k|^4\Big\rangle_k|\eta|^4 + F_{FB} && (16)\\
&= F_n - \alpha\Big(1 - \frac{T}{T_\mathrm{c}}\Big)|\eta|^2 + \frac{\beta}{2}\kappa|\eta|^4, && (17)
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa &= \Big\langle|\hat{d}_k|^4\Big\rangle_k + \gamma - 2\gamma\Big\{\Big\langle|\hat{d}_k^x|^2\Big\rangle_k\Big\}^2 - 2\gamma\Big\{\Big\langle|\hat{d}_k^y|^2\Big\rangle_k\Big\}^2 - 2\gamma\Big\{\Big\langle|\hat{d}_k^z|^2\Big\rangle_k\Big\}^2\\
&\qquad - 4\gamma\Big\{\Big\langle\hat{d}_k^{x*}\hat{d}_k^{y}\Big\rangle_k\Big\}^2 - 4\gamma\Big\{\Big\langle\hat{d}_k^{y*}\hat{d}_k^{z}\Big\rangle_k\Big\}^2 - 4\gamma\Big\{\Big\langle\hat{d}_k^{z*}\hat{d}_k^{x}\Big\rangle_k\Big\}^2. && (18)
\end{aligned}
$$

[1] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975). See Eq. (9.7) therein.

Below T_c, this free energy has a minimum.

$$0 = \frac{\partial F}{\partial \eta^*} = -\alpha\Big(1 - \frac{T}{T_c}\Big)\Big\langle |\hat{d}_k|^2 \Big\rangle_k \eta + \frac{\beta}{2}\kappa 2\eta|\eta|^2, \tag{19}$$

$$\rightarrow \qquad 0 = -\alpha\Big(1 - \frac{T}{T_c}\Big)\Big\langle |\hat{d}_k|^2 \Big\rangle_k + \beta\kappa|\eta|^2. \tag{20}$$

From it,

$$|\eta|^2 = \frac{\alpha}{\beta\kappa}\Big(1 - \frac{T}{T_c}\Big). \tag{21}$$

Substituting this $|\eta|^2$ into the free energy, we obtain the condensation energy F_{cond} as

$$\begin{aligned} F_{cond} &= F - F_n \qquad (22)\\ &= -\alpha\Big(1 - \frac{T}{T_c}\Big)\Big[\frac{\alpha}{\beta\kappa}\Big(1 - \frac{T}{T_c}\Big)\Big] + \frac{\beta}{2}\kappa\Big[\frac{\alpha}{\beta\kappa}\Big(1 - \frac{T}{T_c}\Big)\Big]^2 \qquad (23)\\ &= \frac{-\alpha^2}{2\beta\kappa}\Big(1 - \frac{T}{T_c}\Big)^2. \qquad (24) \end{aligned}$$

10.2 Now, let us compare the condensation energies in the A and B phases of the superfluid ^{3}He on the basis of Eqs. (1) and (2). Assume the following normalized d vectors: the A-phase $\hat{d}_k = \sqrt{\frac{3}{2}}(0, 0, \hat{k}_x + i\hat{k}_y)$, and the B-phase $\hat{d}_k = (\hat{k}_x, \hat{k}_y, \hat{k}_z)$. Here, $\hat{k} = (\hat{k}_x, \hat{k}_y, \hat{k}_z) \equiv \vec{k}/|\vec{k}| = (\cos\phi \sin\theta,\ \sin\phi \sin\theta,\ \cos\theta)$.

a) Let us calculate the factor $\left\langle |\hat{d}_k|^4 \right\rangle_k$, which appears in the condensation energy (Eq. (10)) derived within the weak-coupling BCS theory.

For the A-phase,

$$\left\langle |\hat{d}_k|^4 \right\rangle_k^{(A)} = \left\langle |\hat{d}_k^z|^4 \right\rangle_k \tag{25}$$

$$= \left\langle \left| \sqrt{\frac{3}{2}}(\hat{k}_x + i\hat{k}_y) \right|^4 \right\rangle_k \tag{26}$$

$$= \left(\frac{3}{2}\right)^2 \left\langle \left| (\hat{k}_x + i\hat{k}_y)(\hat{k}_x - i\hat{k}_y) \right|^2 \right\rangle_k \tag{27}$$

$$= \left(\frac{3}{2}\right)^2 \left\langle \left| (\hat{k}_x^2 + \hat{k}_y^2) \right|^2 \right\rangle_k \tag{28}$$

$$= \left(\frac{3}{2}\right)^2 \left\langle \left| \left\{ (\cos\phi \sin\theta)^2 + (\sin\phi \sin\theta)^2 \right\} \right|^2 \right\rangle_k \tag{29}$$

$$= \left(\frac{3}{2}\right)^2 \left\langle \left| \sin^2\theta \right|^2 \right\rangle_k \tag{30}$$

$$= \left(\frac{3}{2}\right)^2 \left\langle \sin^4\theta \right\rangle_k \tag{31}$$

$$= \left(\frac{3}{2}\right)^2 \int \frac{d\Omega_k}{4\pi} \sin^4\theta \tag{32}$$

$$= \left(\frac{3}{2}\right)^2 \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \sin^4\theta \tag{33}$$

$$= \left(\frac{3}{2}\right)^2 \frac{1}{2} \int_0^{\pi} d\theta \sin\theta \sin^4\theta \tag{34}$$

$$= \frac{9}{8} \int_0^{\pi} d\theta \sin^5\theta \tag{35}$$

$$= \frac{9}{8} \cdot \frac{16}{15} \tag{36}$$

$$= \frac{3}{1} \cdot \frac{2}{5} \tag{37}$$

$$= \frac{6}{5}. \tag{38}$$

For the B-phase,

$$\left\langle |\hat{d}_k|^4 \right\rangle_k^{(B)} = \left\langle \left| (|\hat{d}_k|^2) \right|^2 \right\rangle_k \tag{39}$$

$$= \left\langle \left| (|\hat{k}_x|^2 + |\hat{k}_y|^2 + |\hat{k}_z|^2) \right|^2 \right\rangle_k \tag{40}$$

$$= \left\langle \left| 1 \right|^2 \right\rangle_k \tag{41}$$

$$= 1. \tag{42}$$

Therefore, from Eq. (10),

$$\frac{F_{cond}^{(A)}}{F_{cond}^{(B)}} = \frac{\left\langle |\hat{d}_k|^4 \right\rangle_k^{(B)}}{\left\langle |\hat{d}_k|^4 \right\rangle_k^{(A)}} \tag{43}$$

$$= \frac{1}{\left(\frac{6}{5}\right)} \tag{44}$$

$$= \frac{5}{6}. \tag{45}$$

This result indicates that $|F_{cond}^{(B)}| > |F_{cond}^{(A)}|$, namely that within the weak-coupling BCS theory the B phase is energetically more favorable than the A phase.

b) Let us calculate each term of κ in Eq. (18), which appears in the condensation energy (Eq. (24)) where the feedback effect is taken into account.

For the A-phase,

$$\left\langle |\hat{d}_k|^4 \right\rangle_k^{(A)} = \frac{6}{5}. \tag{46}$$

$$\left\langle |\hat{d}_k^x|^2 \right\rangle_k^{(A)} = 0. \tag{47}$$

$$\left\langle |\hat{d}_k^y|^2 \right\rangle_k^{(A)} = 0. \tag{48}$$

$$\left\langle |\hat{d}_k^z|^2 \right\rangle_k^{(A)} = \left\langle \left| \sqrt{\frac{3}{2}} (\hat{k}_x + i\hat{k}_y) \right|^2 \right\rangle_k \tag{49}$$

$$= \left(\frac{3}{2}\right) \left\langle (\hat{k}_x + i\hat{k}_y)(\hat{k}_x - i\hat{k}_y) \right\rangle_k \tag{50}$$

$$= \left(\frac{3}{2}\right) \left\langle (\hat{k}_x^2 + \hat{k}_y^2) \right\rangle_k \tag{51}$$

$$= \left(\frac{3}{2}\right) \left\langle \left\{ (\cos\phi \sin\theta)^2 + (\sin\phi \sin\theta)^2 \right\} \right\rangle_k \tag{52}$$

$$= \left(\frac{3}{2}\right) \left\langle \sin^2\theta \right\rangle_k \tag{53}$$

$$= \left(\frac{3}{2}\right) \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \sin^2\theta \tag{54}$$

$$= \left(\frac{3}{2}\right) \frac{1}{2} \int_0^{\pi} d\theta \sin^3\theta \tag{55}$$

$$= \left(\frac{3}{2}\right) \frac{1}{2} \cdot \frac{4}{3} \tag{56}$$

$$= 1. \tag{57}$$

$$\left\langle \hat{d}_k^{x*}\hat{d}_k^{y}\right\rangle_k^{(A)} = 0. \tag{58}$$

$$\left\langle \hat{d}_k^{y*}\hat{d}_k^{z}\right\rangle_k^{(A)} = 0. \tag{59}$$

$$\left\langle \hat{d}_k^{z*}\hat{d}_k^{x}\right\rangle_k^{(A)} = 0. \tag{60}$$

Therefore, from Eq. (18),

$$\begin{aligned}
\kappa^{(A)} &= \left\langle |\hat{d}_k|^4\right\rangle_k^{(A)} + \gamma - 2\gamma\left\{\left\langle |\hat{d}_k^x|^2\right\rangle_k^{(A)}\right\}^2 - 2\gamma\left\{\left\langle |\hat{d}_k^y|^2\right\rangle_k^{(A)}\right\}^2 - 2\gamma\left\{\left\langle |\hat{d}_k^z|^2\right\rangle_k^{(A)}\right\}^2 \\
&\quad - 4\gamma\left\{\left\langle \hat{d}_k^{x*}\hat{d}_k^{y}\right\rangle_k^{(A)}\right\}^2 - 4\gamma\left\{\left\langle \hat{d}_k^{y*}\hat{d}_k^{z}\right\rangle_k^{(A)}\right\}^2 - 4\gamma\left\{\left\langle \hat{d}_k^{z*}\hat{d}_k^{x}\right\rangle_k^{(A)}\right\}^2 && (61)\\
&= \left\langle |\hat{d}_k|^4\right\rangle_k^{(A)} + \gamma - 2\gamma\left\{\left\langle |\hat{d}_k^z|^2\right\rangle_k^{(A)}\right\}^2 && (62)\\
&= \frac{6}{5} + \gamma - 2\gamma && (63)\\
&= \frac{6}{5} - \gamma. && (64)
\end{aligned}$$

For the B-phase,

$$\left\langle |\hat{d}_k|^4\right\rangle_k^{(B)} = 1. \tag{65}$$

$$\begin{aligned}
\left\langle |\hat{d}_k^x|^2\right\rangle_k^{(B)} &= \left\langle |\cos\phi\sin\theta|^2\right\rangle_k && (66)\\
&= \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \cos^2\phi \sin^2\theta && (67)\\
&= \frac{1}{4\pi}\int_0^{2\pi} d\phi \cos^2\phi \int_0^{\pi} d\theta \sin^3\theta && (68)\\
&= \frac{1}{4\pi}\cdot\pi\cdot\frac{4}{3} && (69)\\
&= \frac{1}{3}. && (70)
\end{aligned}$$

$$\left\langle |\hat{d}_k^y|^2\right\rangle_k^{(B)} = \frac{1}{3}. \tag{71}$$

$$\left\langle |\hat{d}_k^z|^2\right\rangle_k^{(B)} = \frac{1}{3}. \tag{72}$$

$$\begin{aligned}
\left\langle \hat{d}_k^{x*}\hat{d}_k^{y}\right\rangle_k^{(B)} &= \left\langle \hat{k}_x\hat{k}_y\right\rangle_k && (73)\\
&= \left\langle (\cos\phi\sin\theta)(\sin\phi\sin\theta)\right\rangle_k && (74)\\
&= \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta(\cos\phi\sin\theta)(\sin\phi\sin\theta) && (75)\\
&= \frac{1}{4\pi}\int_0^{2\pi} d\phi \cos\phi\sin\phi \int_0^{\pi} d\theta \sin^3\theta && (76)\\
&= 0. && (77)
\end{aligned}$$

$$\left\langle \hat{d}_k^{y*} \hat{d}_k^{z} \right\rangle_k^{(B)} = 0. \tag{78}$$

$$\left\langle \hat{d}_k^{z*} \hat{d}_k^{x} \right\rangle_k^{(B)} = 0. \tag{79}$$

Therefore, from Eq. (18),

$$\begin{aligned}
\kappa^{(B)} &= \left\langle |\hat{d}_k|^4 \right\rangle_k^{(B)} + \gamma - 2\gamma\Big\{\left\langle |\hat{d}_k^x|^2 \right\rangle_k^{(B)}\Big\}^2 - 2\gamma\Big\{\left\langle |\hat{d}_k^y|^2 \right\rangle_k^{(B)}\Big\}^2 - 2\gamma\Big\{\left\langle |\hat{d}_k^z|^2 \right\rangle_k^{(B)}\Big\}^2 \\
&\quad - 4\gamma\Big\{\left\langle \hat{d}_k^{x*} \hat{d}_k^{y} \right\rangle_k^{(B)}\Big\}^2 - 4\gamma\Big\{\left\langle \hat{d}_k^{y*} \hat{d}_k^{z} \right\rangle_k^{(B)}\Big\}^2 - 4\gamma\Big\{\left\langle \hat{d}_k^{z*} \hat{d}_k^{x} \right\rangle_k^{(B)}\Big\}^2 &(80)\\
&= \left\langle |\hat{d}_k|^4 \right\rangle_k^{(B)} + \gamma - 2\gamma\Big\{\left\langle |\hat{d}_k^x|^2 \right\rangle_k^{(B)}\Big\}^2 - 2\gamma\Big\{\left\langle |\hat{d}_k^y|^2 \right\rangle_k^{(B)}\Big\}^2 - 2\gamma\Big\{\left\langle |\hat{d}_k^z|^2 \right\rangle_k^{(B)}\Big\}^2 &(81)\\
&= 1 + \gamma - 2\gamma\Big(\frac{1}{3^2} + \frac{1}{3^2} + \frac{1}{3^2}\Big) &(82)\\
&= 1 + \gamma - \frac{2}{3}\gamma &(83)\\
&= 1 + \frac{1}{3}\gamma. &(84)
\end{aligned}$$

Hence, from Eq. (24),

$$\begin{aligned}
\frac{F_{cond}^{(A)}}{F_{cond}^{(B)}} &= \frac{\kappa^{(B)}}{\kappa^{(A)}} &(85)\\
&= \frac{1 + \frac{1}{3}\gamma}{\frac{6}{5} - \gamma}. &(86)
\end{aligned}$$

Now,

$$1 \equiv \frac{F_{cond}^{(A)}}{F_{cond}^{(B)}} = \frac{1 + \frac{1}{3}\gamma_{\rm c}}{\frac{6}{5} - \gamma_{\rm c}}, \tag{87}$$

$$\rightarrow \qquad 1 + \frac{1}{3}\gamma_{\rm c} = \frac{6}{5} - \gamma_{\rm c}, \tag{88}$$

$$\rightarrow \qquad \frac{4}{3}\gamma_{\rm c} = \frac{1}{5}, \tag{89}$$

$$\rightarrow \qquad \gamma_{\rm c} = \frac{3}{20}. \tag{90}$$

From these results, we notice that $|F_{cond}^{(A)}| > |F_{cond}^{(B)}|$ for $\gamma > \gamma_{\rm c} = 3/20$ (namely, the A-phase is energetically stabilized), while $|F_{cond}^{(B)}| > |F_{cond}^{(A)}|$ for $\gamma < \gamma_{\rm c}$ (namely, the B-phase is stabilized).

Here, we have assumed that γ is a small positive quantity, and then $0 \leq \gamma < \frac{6}{5}$.