Unkonventionelle Supraleitung

Serie 11

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11 Let us consider the superconductivity in a system with tetragonal crystal structure and strong spin-orbit coupling discussed in the theory lecture notes. For superconducting phases described by two-component order parameter $\vec{\eta} = (\eta_x, \eta_y)$ corresponding to two basis functions of the two dimensional representation [Eq. (4.20) or (166) in the theory lecture notes], the Ginzburg-Landau free energy density f *in the spatially uniform case under zero magnetic field* is given as follows [see Eq. (4.21) or (167)].

$$f = a(T)|\vec{\eta}|^2 + b_1|\vec{\eta}|^4 + \frac{b_2}{2}\{\eta_x^{*2}\eta_y^2 + \eta_x^2\eta_y^{*2}\} + b_3|\eta_x|^2|\eta_y|^2.$$

Here, we have omitted the gradient terms and the magnetic field term because of the spatially uniformity assumed. The coefficients $a(T)$ and b_i $(i = 1, 2, 3)$ are real numbers. $a(T) < 0$ for $T < T_c$.

a) Derive the following two coupled Ginzburg-Landau equations by taking the variation of f with respect to η_x^* $(\partial f/\partial \eta_x^* = 0)$ and η_y^* $(\partial f/\partial \eta_y^* = 0)$.

$$\begin{aligned} a(T)\eta_x + 2b_1\Big\{|\eta_x|^2 + |\eta_y|^2\Big\}\eta_x + b_2\eta_y^2\eta_x^* + b_3|\eta_y|^2\eta_x &= 0, \\ a(T)\eta_y + 2b_1\Big\{|\eta_y|^2 + |\eta_x|^2\Big\}\eta_y + b_2\eta_x^2\eta_y^* + b_3|\eta_x|^2\eta_y &= 0. \end{aligned}$$

b) Let us parameterize the order parameters η_x and η_y as

$$(\eta_x, \eta_y) = (\eta_0 \cos\alpha,\ \eta_0 e^{i\gamma}\sin\alpha),$$

with η_0 real, α $(-\pi/2 < \alpha \le \pi/2)$, and γ $(0 \le \gamma < 2\pi)$.

Derive the following expression for the Ginzburg-Landau free energy density f.

$$f = a(T)\eta_0^2 + \frac{1}{4}\Big[4b_1 + \sin^2 2\alpha(b_2\cos 2\gamma + b_3)\Big]\eta_0^4.$$

c) Because $a(T) < 0$, this Ginzburg-Landau free energy density f has a minimum with respect to η_0 when the coefficient of the second term is positive, namely when

$$4b_1 + \sin^2 2\alpha(b_2\cos 2\gamma + b_3) > 0.$$

Show that η_0 which yields a minimum of f is given by

$$\eta_0^2 = \frac{-2a(T)}{4b_1 + \sin^2 2\alpha(b_2\cos 2\gamma + b_3)}.$$

Show also that the minimum value of f with respect to η_0 is given by

$$f = \frac{-\{a(T)\}^2}{4b_1 + \sin^2 2\alpha(b_2\cos 2\gamma + b_3)}.$$

From now on, let us assume $b_1 > 0$.

d) Next, let us minimize f with respect to the parameter α. The free energy density f in the Problem **c)** has a minimum when its denominator (> 0) is minimized.

Show the following by considering the sign of the factor $(b_2 \cos 2\gamma + b_3)$ and the value of α.

$$(\eta_x, \eta_y) = \begin{cases} \left(\frac{\eta_0}{\sqrt{2}},\ \pm\frac{\eta_0}{\sqrt{2}} e^{i\gamma}\right) & (b_2 \cos 2\gamma + b_3 < 0, \quad 4b_1 + b_2 \cos 2\gamma + b_3 > 0) \\ (\eta_0, 0) \quad \text{or} \quad (0, \eta_0 e^{i\gamma}) & (b_2 \cos 2\gamma + b_3 > 0, \quad 4b_1 > 0) \end{cases}$$

Here, take account of also the condition, $4b_1 + \sin^2 2\alpha (b_2 \cos 2\gamma + b_3) > 0$, mentioned in the Problem **c)**.

e) Finally, let us minimize f with respect to the parameter γ.

Show the following by considering the result of the Problem **d)**, the factor $(b_2 \cos 2\gamma)$ in the denominator of f, and the sign of b_2.

$$(\eta_x, \eta_y) = \begin{cases} \left(\frac{\eta_0}{\sqrt{2}},\ \pm i\frac{\eta_0}{\sqrt{2}}\right) & (-b_2 + b_3 < 0, \quad b_2 > 0, \quad 4b_1 - b_2 + b_3 > 0) \\ \left(\frac{\eta_0}{\sqrt{2}},\ \pm\frac{\eta_0}{\sqrt{2}}\right) & (b_2 + b_3 < 0, \quad b_2 < 0, \quad 4b_1 + b_2 + b_3 > 0) \\ (\eta_0, 0) \quad \text{or} \quad (0, \eta_0 e^{i\gamma}) & (b_2 \cos 2\gamma + b_3 > 0, \quad b_1 > 0) \end{cases}$$

Comment: The superconducting phases represented by the order parameters in the last equation above correspond to the phases A, B, and C discussed in the theory lecture notes (see Fig. 7.1 or Fig. 10 therein). The phases A, B, and C correspond to the order parameters from top to bottom in the last equation above, respectively. From the above results, one can easily confirm that the free energy density ($f < 0$) in the Problem **c)** is lower for the phases A and B [corresponding to the first (A) and second (B) lines in the last equation] than for the phase C [the third line]. (Consider the denominator of f, the value of α, and the sign of the factor $(b_2 \cos 2\gamma + b_3)$ for each phase.) Therefore, concerning the phase diagram in the (b_2, b_3) parameter space, the phase A or B is energetically more favorable than the C phase in the region where the inequalities in the first or second line in the last equation are satisfied.