Unkonventionelle Supraleitung

Serie 2

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2.1 Assuming that liquid ^{3}He may be described as a Fermi *gas* (FG), with a molar volume of 37 cm^3, estimate the following:
i) The Fermi temperature T_{F}, the magnetic susceptibility χ_{FG}, the specific heat C_{FG}, and the sound velocity S_{FG}.

Then, to appreciate the deviations of the actual properties of ^{3}He (Fermi *liquid*) from the expectations for a Fermi *gas*, compare your estimates with the experimental low-temperature (between 2 and 100 mK) values, given in the following table:

Low-Temperature Properties of ^{3}He at Atmospheric Pressure

C/RT (K^{-1})	S (ms^{-1})	χ (cgs)	KT (K·μW/m)	ηT^2 (poise K^2)
2.78	183	$3.3\times10^{38}\beta^2$	500	2.5×10^{-6}

Here K is the thermal conductivity, η is the viscosity, β is the magnetic moment of the ^{3}He nucleus in cgs units, and $R = N_{\mathrm{A}}k_{\mathrm{B}}$ is the gas constant (with N_{A} the Avogadro number: $N_{\mathrm{A}} = 6.022 \times 10^{23}$, and k_{B} the Boltzmann constant: $k_{\mathrm{B}} = 1.38 \times 10^{-16}$ erg·K^{-1}).

Hints: Recall the following (refer to the lecture notes of Solid State I, or see standard textbooks on the solid state physics, such as, *"Introduction to Solid State Physics"* by C. Kittel)

- The density of ^{3}He is $\rho = N/V = N_{\mathrm{A}}/37\mathrm{cm}^3$ and the bare mass of ^{3}He nucleus is $m_3 = 0.5008 \times 10^{-23}$ g.
- The Fermi wave number is $k_{\mathrm{F}} = (3\pi^2\rho)^{1/3}$. The Fermi energy is $E_{\mathrm{F}} = \hbar^2 k_{\mathrm{F}}^2/2m_3$. $T_{\mathrm{F}} = E_{\mathrm{F}}/k_{\mathrm{B}}$. The density of states is $N(E_{\mathrm{F}}) = 2D(E_{\mathrm{F}}) = 3\rho/2E_{\mathrm{F}}$ [$D(E_{\mathrm{F}})$ is the density of states *per spin*]. $\hbar = 1.055 \times 10^{-27}$ erg s.
- χ_{FG} and C_{FG} for a Fermi gas, may also be obtained from Eqs. II.3, II.4, II.5, and II.13 of the experiment lecture notes, setting the Fermi-liquid parameters F_l and Z_l to zero and replacing the magnetic moment μ_{B} of a free electron by the ^{3}He nuclear magnetic moment β, and $m^* \to m_3$.
- When calculating C_{FG}/RT, use $N(E_{\mathrm{F}})$ *per mol* defined as $N(E_{\mathrm{F}}) = 3N_{\mathrm{A}}/2E_{\mathrm{F}}$.
- $S_{\mathrm{FG}}^2 = 1/(\kappa m_3 \rho)$, where κ is the compressibility [Use the expression for κ^{-1} shown in the problem **1.1** or Eq. II.8 in the experiment lecture notes, and note the chemical potential $\mu \approx E_{\mathrm{F}}$ at sufficiently low temperatures].

ii) How well localized in real space are the ^{3}He atoms? (Provide a rough estimate of the uncertainty in the position of the ^{3}He atoms in any direction, in units of Å.)

Hint: Consider the Heisenberg's uncertainty relation $\Delta x \Delta p \sim h$, ($p = \hbar k$), or consider the wavelength of the de Broglie wave for a ^{3}He atom with momentum $p = (2m_3 k_{\mathrm{B}} T_{\mathrm{F}})^{1/2}$.

2.2 Considering liquid ^{3}He to be a Fermi *liquid* (FL).
i) What is the value of the effective mass (m_3^*/m_3) of the ^{3}He atoms?
ii) Estimate the Fermi-liquid parameters Z_0 and F_1 for ^{3}He.
iii) Compare the observed values of K and η, at 2 mK and atmospheric pressure (see the above table in **2.1**), with those of other familiar liquids such as oil and water at 300 K, N_2 at 77 K, and ^{4}He at 4.2 K.‡

Hints: For the comparison of the Fermi liquid (FL) and Fermi gas (FG) properties recall that

- The specific heat C: $C \propto N(E_\mathrm{F})$ and $N(E_\mathrm{F}) \propto 1/E_\mathrm{F} \propto m$. Then, consider the ratio $C_\mathrm{exp}/C_\mathrm{FG}$. ($C_\mathrm{exp}$ is the experimental value given in the table of Probl. **2.1**.)
- Refer to Eq. II.5 in the experiment lecture notes, for the effective mass $m_{FL} = m_3^*$ as a function of the bare mass, $m_{FG} = m_3$, and the Fermi-liquid parameter F_1. Also, refer to Eq. II.13 for the magnetic susceptibility χ_{FL}, in which Z_0 appears. Note $\chi \propto N(E_\mathrm{F}) \propto m$. And $k_\mathrm{F} = (3\pi^2\rho)^{1/3}$ is a function of only the density ρ and therefore k_F is the same for both the FG and the FL.
- From the table of **2.1**, at $T = 2$ mK, the thermal conductivity $K = 500/(2 \times 10^{-3})$ [μW/m] and the viscosity $\eta = 2.5 \times 10^{-6}/(2 \times 10^{-3})^2$ [poise].

‡ Sources: For the properties of common liquids see: "Properties of Materials at low temperatures, a compendium", V.J. Johnson (Ed) NBS/USA 1961. For the properties of normal liquid ^{3}He see: "The theory of Quantum Liquids" Vol. I, Chap. I, by D. Pines and P. Nozières, Addison-Wesley, 1966 (see references therein).