Unkonventionelle Supraleitung

Serie 5

Verteilung: 29.November Abgabe: 6.Dezember

For triplet states, the order parameter is written as

$$\hat{\Delta}_k = \vec{d}_k \cdot \hat{\vec{\sigma}} i\hat{\sigma}_y = \begin{pmatrix} -d_k^x + id_k^y & d_k^z \\ d_k^z & d_k^x + id_k^y \end{pmatrix},$$

where $\vec{d}_k = (d_k^x, d_k^y, d_k^z)$ is the d vector, and $\hat{\vec{\sigma}} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ are the Pauli matrices.

5.1 Consider the expectation value of the spin, $\langle \vec{S} \rangle$, defined by

$$\langle \vec{S} \rangle \equiv \frac{\int \frac{d\Omega_k}{4\pi} \mathrm{Tr}[\hat{\Delta}_k^\dagger \vec{S} \hat{\Delta}_k]}{\int \frac{d\Omega_k}{4\pi} \mathrm{Tr}[\hat{\Delta}_k^\dagger \hat{\Delta}_k]},$$

where $\vec{S} = \frac{\hbar}{2}\hat{\vec{\sigma}}$.

a) Show that $\langle \vec{S} \rangle$ is expressed for triplet states as

$$\langle \vec{S} \rangle = -i\frac{\hbar}{2} \int \frac{d\Omega_k}{4\pi} \hat{d}_k^* \times \hat{d}_k.$$

Here, $\hat{d}_k \equiv \vec{d}_k / |\vec{d}|$, and $|\vec{d}| = \sqrt{|\vec{d}|^2}$ is defined as $|\vec{d}|^2 \equiv \int \frac{d\Omega_k}{4\pi} \frac{1}{2} \mathrm{Tr}\left[\hat{\Delta}_k^\dagger \hat{\Delta}_k\right]$.

b) Calculate $\langle \vec{S} \rangle$ for the following three phases of ^{3}He: the A-phase $\hat{d}_k = \sqrt{\frac{3}{2}}(0, 0, \hat{k}_x + i\hat{k}_y)$, the A$_1$-phase $\hat{d}_k = \sqrt{\frac{3}{4}}\left(\hat{k}_y + i\hat{k}_z, i(\hat{k}_y + i\hat{k}_z), 0\right)$, and the B-phase $\hat{d}_k = (\hat{k}_x, \hat{k}_y, \hat{k}_z)$. Here, $\hat{k} = (\hat{k}_x, \hat{k}_y, \hat{k}_z) \equiv \vec{k}/|\vec{k}|$.

5.2 Consider the expectation value of the orbital angular momentum, $\langle \vec{L} \rangle$, defined by

$$\langle \vec{L} \rangle \equiv \frac{\int \frac{d\Omega_k}{4\pi} \mathrm{Tr}[\hat{\Delta}_k^\dagger \vec{L} \hat{\Delta}_k]}{\int \frac{d\Omega_k}{4\pi} \mathrm{Tr}[\hat{\Delta}_k^\dagger \hat{\Delta}_k]},$$

where $\vec{L} = -i\hbar(\hat{k} \times \vec{\nabla}_{\hat{k}})$, and $\vec{\nabla}_{\hat{k}} = (\frac{\partial}{\partial \hat{k}_x}, \frac{\partial}{\partial \hat{k}_y}, \frac{\partial}{\partial \hat{k}_z})$.

a) Show that $\langle \vec{L} \rangle$ is expressed for triplet states as

$$\langle \vec{L} \rangle = -i\hbar \int \frac{d\Omega_k}{4\pi} \hat{d}_k^* \cdot [(\hat{k} \times \vec{\nabla}_{\hat{k}})\hat{d}_k].$$

b) Calculate $\langle \vec{L} \rangle$ for the above phases of ^{3}He: the A-phase, the A$_1$-phase, and the B-phase.

Hint: One may utilize the formulas:

$$\int_0^\pi d\theta \sin^3\theta = \frac{4}{3}, \qquad \int_0^\pi d\theta \sin\theta\cos^2\theta = \frac{2}{3}.$$