Unkonventionelle Supraleitung

Serie 6

Verteilung: 6.Dezember Abgabe: 13.Dezember

The density of states per both spin projections and per unit volume is defined as

$$N(E) \;=\; \frac{1}{V}\sum_k \delta(E_k - E),$$

where E_k is the quasiparticle energy spectrum in the superconducting state, and V is the volume of the system. For the spin-triplet unitary states (and the spin-singlet states), E_k (≥ 0) is given by

$$E_k \;=\; \sqrt{\xi_k^2 + |\Delta_k|^2}, \qquad \text{with} \qquad |\Delta_k|^2 = \frac{1}{2}\mathrm{Tr}[\hat{\Delta}_k \hat{\Delta}_k^\dagger],$$

where ξ_k is the energy spectrum in the normal state, ($\xi_k = 0$ at the Fermi level). Then, $N(E)$ (for $E \geq 0$) is

$$\begin{aligned} N(E) \;&=\; \frac{1}{V}\sum_k \delta(\sqrt{\xi_k^2 + |\Delta_k|^2} - E) \\ &=\; \int \frac{d\Omega_k}{4\pi}\int_{-\infty}^{\infty} d\xi_k D(\xi_k)\delta(\sqrt{\xi_k^2 + |\Delta_k|^2} - E) \\ &\approx\; D(0)\int \frac{d\Omega_k}{4\pi}\int_{-\infty}^{\infty} d\xi_k \delta(\sqrt{\xi_k^2 + |\Delta_k|^2} - E) \\ &=\; \frac{N_0}{2}\int \frac{d\Omega_k}{4\pi}\int_{-\infty}^{\infty} d\xi_k \delta(\sqrt{\xi_k^2 + |\Delta_k|^2} - E) \\ &=\; N_0\int \frac{d\Omega_k}{4\pi}\int_{0}^{\infty} d\xi_k \delta(\sqrt{\xi_k^2 + |\Delta_k|^2} - E), \end{aligned}$$

where N_0 is the density of states in the normal state at the Fermi level per both spin projections and per unit volume. ($D(\xi_k)$ is the density of states in the normal state *per spin projection* and per unit volume. In the above integrand, only the vicinity of the Fermi surface ($\xi_k \sim 0$) is important, because we are interested in the low-energy excitations E. Therefore, we have approximated $D(\xi_k) \approx D(0)$.)

6.1 Show that $N(E)$ (for $E \geq 0$) is written as

$$N(E) \;=\; N_0 \int \frac{d\Omega_k}{4\pi}\mathrm{Re}\frac{E}{\sqrt{E^2 - |\Delta_k|^2}}.$$

Here, “Re” means taking the real part.

<u>Hint</u>: Note that $\xi_k^2 = E_k^2 - |\Delta_k|^2$ and ξ_k is a real number.

6.2 Show that for the following three spin-triplet unitary states, the density of states $N(E)$ (for $E \geq 0$) is given as follows. ($\hat{k} = (\hat{k}_x, \hat{k}_y, \hat{k}_z) = \vec{k}/|\vec{k}|$)

(1) For the ABM state $\vec{d}_k = \Delta_0(0, 0, \hat{k}_x + i\hat{k}_y)$,

$$N(E) = N_0 \frac{E}{2|\Delta_0|} \ln\left|\frac{E + |\Delta_0|}{E - |\Delta_0|}\right|.$$

(2) For the BW state $\vec{d}_k = \Delta_0(\hat{k}_x, \hat{k}_y, \hat{k}_z)$,

$$N(E) = \begin{cases} N_0 \dfrac{E}{\sqrt{E^2 - |\Delta_0|^2}} & (E > |\Delta_0|) \\ 0 & (0 \leq E < |\Delta_0|) \end{cases}$$

(3) For the polar state $\vec{d}_k = \Delta_0(0, 0, \hat{k}_z)$,

$$N(E) = \begin{cases} N_0 \dfrac{E}{|\Delta_0|} \arcsin\left(\dfrac{|\Delta_0|}{E}\right) & (E > |\Delta_0|) \\ N_0 \dfrac{E}{|\Delta_0|} \dfrac{\pi}{2} & (0 \leq E < |\Delta_0|) \end{cases}$$

Hint: One may utilize the formulas:

$$\int dt \frac{1}{\sqrt{t^2 + a}} = \ln\left|\left(t + \sqrt{t^2 + a}\right)\right|,$$

$$\int dt \frac{1}{\sqrt{b^2 - t^2}} = \arctan\left(\frac{t}{\sqrt{b^2 - t^2}}\right). \qquad \left(|b| \geq |t|, \quad -\frac{\pi}{2} \leq \arctan(\cdots) \leq \frac{\pi}{2}\right)$$